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Environmental Taxation and Strategic Commitment in Duopoly Models

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Abstract. In this paper, we address the issue of optimal environmental taxation under imperfect competition. The problem is analysed for three different types of duopoly models, the Cournot open and closed loop models, and the Stackelberg model. We explicitly analyse the role of strategic behaviour. Each firm has to make a choice of output level and of the level of a strategic variable. The choice of this strategic variable affects both marginal cost and emissions. We compare the properties of these three duopoly models, and derive and compare optimal environmental taxes. We show that whether the optimal tax is lower or higher than marginal environmental costs depends on the information transmission and the effect of the strategic variable on marginal costs. In addition, the differences in market shares, and the influence of the tax on the cost structure play important roles, in determining optimal emission taxes.

Key words: environmental taxation, oligopoly, optimal taxation, strategic commitment

JEL classification: D62, L13

1. Introduction

Many markets consist of only a few firms. At the same time many of the policy recommendations in environmental economics are based on the assumption of perfect competition. For example under perfect competition the optimal environmental tax is equal to the marginal damage cost, i.e. a Pigouvian tax. Since many markets are not perfectly competitive it is of general interest to analyse optimal environmental taxes under imperfect competition. The problem of optimal environmental taxation under imperfect competition consists of two parts, optimal level of abatement, and optimal correction of imperfect competition distortions. The problem was first analysed in Buchanan (1969) and in Barnett (1980), they show that for an externality produced by a monopolist, the optimal tax is lower than marginal environmental costs. Simpson (1995) investigates the case of an optimal environmental tax in a Cournot duopoly, and he shows that the optimal tax is not necessarily lower than marginal environmental costs. The reason for this is that a Cournot duopoly might result in inefficient allocations of production between firms, and if a higher tax shifts production to the more efficient firm then it might be optimal to set the tax higher. Further, Katsoulacos and Xepapadeas (1995) show that with an endogenous market structure the optimal tax might exceed the marginal environmental cost, since there might be a negative welfare effect of too many entrants.[1] In this paper we compare the properties of three duopoly models with exogenous market structure, and we derive optimal environmental taxes for all models. In these models, each firm has to make a choice of output and strategic variable, where the strategic variable affects both production costs and emissions. The interesting aspect of these models is the role of the strategic variable, which has not been considered in the previously literature.[2] For example, suppose a firm invests in capital that reduces the emissions per unit of output. Given an emission tax this will reduce the marginal costs. At the same time, this investment can affect other types of production costs, including abatement costs, and these effects can be both positive and negative. Furthermore, the investment can affect the market equilibrium, which also affect the emissions. Consequently, several effects are important for the optimal emission tax. Not even the effect on emissions of a tax is clear, since the tax can result in shifts of production between firms, and strategic considerations can result in increased production. As is well known in the literature, the effects of strategic commitment depend crucially on the information transmission. Therefore, we consider three duopoly models, Cournot open and closed loop, and Stackelberg, all with a different information transmission structure. The aim of the paper is to derive optimal tax expressions for different oligopoly models under general specifications of the effect of the strategic variable. Furthermore, we investigate the effects of the emission tax, and the possibility of perverse effects on the emissions of the tax. We will also derive conditions under which the optimal tax is lower or higher than marginal environmental costs, and show that the optimal tax expressions are sensitive to the underlying assumptions about cost- and demand-conditions, as well as the presence of strategic effects.

2. The Model

We assume that the regulator's only instrument is an emission tax, and that he cannot differentiate the per emission-unit tax between the two firms. The regulation game is conducted in three stages. In stage 1, the regulator announces the tax scheme. In stage 2, each firm chooses its level of the strategic variable. Finally, in stage 3, each firm chooses its level of output. The models considered are the Cournot-Nash duopoly open and closed loop, and the Stackelberg-Nash duopoly. The reason for this is that we wish to consider the effect of strategic behaviour on the environmental tax. The difference between these models is the information transmission. In the Cournot open loop model, firms simultaneously choose their levels of the strategic variable and output. In the Cournot closed loop model, the firms first simultaneously choose their levels of the strategic variable, and then simultaneously choose their output levels. In the Stackelberg model, the leader first chooses his level of the strategic variable, and then the follower decides his level.

The firms then simultaneously choose their output levels. The strategic variable in stage 2, could for example be investments in abatement capital, R&D, or simply reflect different technologies. For simplicity, we call the strategic variable capital.

Both firms can invest in capital, x^i, at cost r per unit, and we assume that they cannot change their level of capital in the third stage. The profit function for firm i is:

$$\Pi^i(Q, x^i, t) = R^i(q^i, Q^j) - C^i(q^i, x^i) - rx^i - te^i(q^i, x^i);\ i, j = 1, 2 \text{ and } i \neq j, \tag{1}$$

where $R^i = P(Q)q^i$, $Q = q^1 + q^2$, $e^i_x < 0$, $e^i_{xx} > 0$, $e^i_q > 0$. Emissions from firm i are e^i, and the emission tax per unit of emission is t. In standard models of strategic investments, marginal costs are decreasing in the strategic variable. Otherwise, the firms would not have any incentives for investments. However, in this model, investments in capital affect both marginal production costs and emissions. Total marginal cost is the sum of marginal production cost and marginal tax cost, $c^i = C^i_{q^i} + te^i_{q^i}$. We assume that total marginal cost is either increasing or decreasing in capital at the equilibrium level of output, i.e. $c^i_x > 0$ or $c^i_x < 0$, since it is not clear that for example investment in abatement technology reduces total marginal costs. This is an important aspect of this paper, and it has not been considered in previous models. However, in oligopoly models focusing on the trade-off between commitment and flexibility, marginal costs can, in principle, be increasing in strategic investments (e.g. Spencer and Brander 1992; Vives 1989). Carlsson (1999) uses the following cost function that allows for both cases:

$$C^i = \theta(x^i)q^i t + \gamma(x^i)(q^i)^2/2 + rk^i;\ \theta(x^i) > 0, \theta'(x^i) < 0, \gamma(x^i) > 0, \tag{2}$$

where $\theta(x^i)q^i$ are emissions from firm i, and the sign of $\gamma'(x^i)$ represents the firm's flexibility of capital investments. Capital investment would then shift down the intercept of the marginal cost curve because of reduced emissions. If $\gamma'(x^i) < 0$, usually referred to flexibility-increasing investments, then marginal costs would always be decreasing in capital. However, if $\gamma'(x^i) > 0$, flexibility-reducing investments, the effect on the equilibrium marginal cost is not determined. In the paper we do not use a specific functional form for the cost function, but the function above would be an obvious candidate.

Define a symmetric equilibrium (SE) as an equilibrium where $q^{iN}(x^i, t) = q^{jN}(x^j, t)$, where $x^i(t) = x^j(t)$, and $x^{iN} = x^{jN}(t)$. Further, we impose the following restrictions on the profit function:

$$|\Pi^i_{ii}| > |\Pi^i_{ij}| \text{ and } \Pi^i_{ij} < 0 \Rightarrow \rho^i \in (-1, 0);\ \rho^i = dq^{iN}/dq^j = -\Pi^i_{ij}/\Pi^i_{ii}$$
$$|\Gamma^i_{ii}| > |\Gamma^i_{ij}| \Rightarrow \Gamma^i_{ii}\Gamma^i_{jj} - \Gamma^i_{ij}\Gamma^j_{ji} > 0 \text{ and } \gamma^i \in (-1, 1);\ \gamma^i = dx^{iN}/dx^j = -\Gamma^i_{ij}/\Gamma^i_{ii}, \tag{3}$$

where Γ^i_i and Γ^i_i are partial derivatives of the profit function with respect to output and capital respectively, and N denotes the Nash equilibrium levels of output and capital. The slope of firm *i*'s reaction curve in output and capital space is denoted ρ^i and γ^i respectively. Consequently, we assume that firms' outputs are strategic substitutes.[3] Further, we distinguish between the cases where firms' investments in capital are (i) strategic substitutes, $\gamma^i \in (-1,0)$, and $\Gamma^i_{ij} < 0$, and (ii) strategic complements, $\gamma^i \in (0,1)$, and $\Gamma^i_{ij} > 0$. The regulator maximises the unweighted sum of consumer and producer surplus, fixed capital costs, and external costs of emissions:

$$W = \int_0^Q P(z)dz - C^1 - C^2 - rx^1 - rx^2 - D(e^1 + e^2), \tag{4}$$

where *P* is the inverse demand function, and *D* is social cost of emissions. We assume that *D* is strictly increasing in e^i.

3. The Optimal Environmental Tax

In order to derive an optimal tax expression we must first solve, by backward induction, the firms' profit maximising problem. After that we can solve the regulator's welfare maximising problem. We derive separate optimal tax expressions for the three models.

In the Cournot open loop, stage 2 and 3 are played simultaneously, and therefore each firm maximises its profit function with respect to q^i and x^i, given its rivals' capital and output. First-order conditions are therefore:

$$\Pi^i_i(q^i, q^j) = R^i_i - c^i = 0, \text{ and} \tag{5}$$

$$\Gamma^i_i(x^i, x^j) = -C^i_x - r - te^i_x = 0, \tag{6}$$

where subscripts denote partial derivatives. In stage 1, the regulator maximises the welfare function with respect to the emission tax, under the restriction that the firms maximise their profits. Therefore, we differentiate $W(x^{1N}, x^{2N}, t)$ with respect to *t*, and use the first order conditions (5) and (6) to arrive at:

$$t = \frac{\partial D}{\partial e} = \frac{P'q^{1N}\frac{dq^{1N}}{dt} + P'q^{2N}\frac{dq^{2N}}{dt}}{de^1/dt + de^2/dt}. \tag{7}$$

For a proof, see Appendix. The optimal tax expression has two parts, the first corresponds to the standard Pigouvian tax, i.e. marginal environmental costs, and the second is a correction due to imperfect competition. This is a standard result for optimal emission taxes under imperfect competition (see e.g. Simpson 1995).[4] The interesting question is what sign the second part of the expression has, since this determines whether the tax should be lower or higher than marginal environmental costs. However, before investigating this, we derive optimal taxes for the other models.

For the Cournot closed loop, in stage 3, each firm maximises its profit function with respect to q^i, taking its rival's capital and output as given. In stage 2, each firm maximises its profit function with respect to x^i given its rival's level of x^j. The first-order conditions are therefore (5) and:

$$\Gamma_i^i(x^i, x^j) = R_j^i \frac{\partial q^j}{\partial x^i} - C_x^i - r - te_x^i = 0. \tag{8}$$

In stage 1, the procedure is the same as in the open loop case. Thus, the optimal tax is:

$$t = \frac{\partial D}{\partial e} + \frac{P' q^{1N}\left(\frac{dq^{1N}}{dt} + \frac{\partial x^{2N}}{\partial x^{1N}}\frac{dx^{1N}}{dt}\right) + P' q^{2N}\left(\frac{dq^{2N}}{dt} + \frac{\partial x^{1N}}{\partial x^{2N}}\frac{dx^{2N}}{dt}\right)}{de^1/dt + de^2/dt}. \tag{9}$$

Again, the optimal tax has two parts, but the second part has an additional term compared with the open loop. We denote this term the strategic effect, i.e. the effect of the rival's capital on output.

For the Stackelberg model, let firm 1 be leader and firm 2 follower. The leader first chooses his level of capital, and then the follower chooses his level. Then, the firms choose their levels of output simultaneously. The first-order conditions are (5), and:

$$\Gamma_1^1 = R_2^1 \frac{\partial q^2}{\partial x^2}\frac{dx^2}{dx^1} + R_2^1 \frac{\partial q^2}{\partial x^1} - C_x^1 - r - te_x^1 = 0 \text{ and } \Gamma_2^2 = R_1^2 \frac{\partial q^1}{\partial x^2} - C_x^2 - r - te_x^2 = 0. \tag{10}$$

In stage 1, the procedure is the same as in the Cournot cases. Thus, the optimal tax is

$$t = \frac{\partial D}{\partial e} + \frac{P' q^{1N}\left(\frac{dq^{1N}}{dt} + \frac{\partial x^{2N}}{\partial x^{1N}}\frac{dx^{1N}}{dt} + \frac{\partial x^{2N}}{\partial x^{2N}}\gamma^2\frac{dx^{1N}}{dt}\right) + P' q^{2N}\left(\frac{dq^{2N}}{dt} + \frac{\partial x^{1N}}{\partial x^{2N}}\frac{dx^{2N}}{dt}\right)}{de^1/dt + de^2/dt}. \tag{11}$$

Again, the second part of the optimal tax expression has an additional term. We denote this term the leader effect, i.e. the effect of the leader's capital on the follower's capital.

4. The Effects of the Environmental Tax and Strategic Behaviour

In this section we investigate the effects of the emission tax on equilibrium levels of capital, output and emissions. Though the exercises are tedious, they enable us to analyse the sign and magnitude of the second part of the optimal tax expressions. The analysis of the Cournot closed loop case easily applies to the Stackelberg case, and therefore we focus on the two Cournot cases.

4.1. The Effect of the Tax on the Level of Capital

Differentiating (6) with respect to the tax, it is easy to show that for the open loop model both firms' levels of capital are increasing in the emission tax. For the closed loop model, differentiating (8) with respect to the tax and rearranging, we have:

$$\frac{dx^{1N}}{dt} + \frac{dx^{2N}}{dt} > 0 \text{ and } \frac{dx^{iN}}{dt} = \frac{e_x^i(\Gamma_{ii}^i)^{-1} + \gamma^i e_x^j(\Gamma_{jj}^j)^{-1}}{1 - \gamma^i\gamma^j} > 0 \text{ if } \Gamma_{ij}^i > 0. \tag{12}$$

For a proof, see Appendix. Thus for strategic complements, both firms' levels of capital are increasing in the tax. For strategic substitutes, the effect on firm's level of capital is not clearly determined, but industry level of capital is increasing in the tax. However, for a SE, both firms' levels of capital are increasing in the tax, even for strategic substitutes. From (12) we have that whether or not a firm's capital is decreasing in the tax depends on the effects on the firms' emissions and marginal profit functions:

$$dx^{iN}/dt < 0 \text{ iff } dx^{jN}/dt > 0 \text{ and } e_x^i/e_x^j < \Gamma_{ij}^i/\Gamma_{jj}^j. \tag{13}$$

A firm's capital is decreasing in the tax if the increase in emissions is small, the decrease in the rival's emissions is large, and the cross effect of the rival's capital on marginal profit exceeds the own effect. The last effect can also be expressed in terms of the slope of the reaction curves. If the cross effect, $|\Gamma_{ij}^i|$, is large, firm i is sensitive to a change in the rival's capital, and if the rival's own effect, $|\Gamma_{jj}^j|$, is small, the rival is sensitive to a change in firm i's capital.

4.2. THE EFFECT OF THE ENVIRONMENTAL TAX ON OUTPUT

The effect on output is somewhat more complicated. For the open loop model, industry output is decreasing in the tax when marginal costs are increasing in capital, and increasing when marginal costs are decreasing in capital. However, the effect on the individual firm's output is not determined. This is seen by differentiating (5) with respect to the tax, at the equilibrium levels of capital:

$$\frac{dq^{iN}}{dt} = \frac{c_x^i(dx^{iN}/dt)(\Pi_{ii}^i)^{-1} + \rho^i c_x^j(dx^{jN}/dt)(\Pi_{jj}^j)^{-1}}{1-\rho^i\rho^j} \text{ and } \frac{dQ^N}{dt} < (>)0 \text{ iff } c_x^i > (<)0;$$
$$\Rightarrow \forall i c_x^i > (<)0 : \frac{dq^{iN}}{dt} < (>)0 \text{ iff } \frac{dq^{jN}}{dt} < (>)0 \text{ and } \left|c_x^i \frac{dx^{iN}}{dt}\right| < \left|c_x^j \frac{dx^{jN}}{dt}\frac{\Pi_{ij}^i}{\Pi_{jj}^j}\right|. \tag{14}$$

For a proof, see Appendix. For simplicity, we restrict our attention to the case where marginal costs are increasing in capital. Output of at least one firm must then be decreasing in the tax, and for a SE output of both firms must be decreasing in the tax. The effect on the individual firm depends on the effect of the tax on both firms' costs, and the effect of the rival's capital on both firms' marginal profit functions. The disparity in the cost imposed by the tax depends on the effects on marginal costs of increased capital and on the effects on capital of the tax. For example, firm i's output is increasing in the tax if the increase in firm i's marginal cost is small, the increase in the rival's marginal cost is large, and both firms are sensitive to a change in the rival's output.

For the closed loop model, differentiating (5), at the equilibrium levels of capital, with respect to the tax we have that

$$\frac{dQ^N}{dt} < (>)0 \text{ if } \forall i \left(\frac{dx^{iN}}{dt} > 0 \text{ and } c_x^i > (<)0\right). \tag{15}$$

The effect on industry output is only determined when both firms' levels of capital are increasing in the tax, and in that case the analysis of the open loop for the effects on output applies.[5] Therefore, we restrict our attention to strategic substitutes and the case when firm 1's capital is decreasing in the tax, and consequently firm 2's capital is increasing in the tax. From (14) we find that:

$$\frac{dx^{1N}}{dt} < 0 \text{ and } \forall i c_x^i > (<)0 \Rightarrow \frac{dq^{1N}}{dt} > (<)0 \text{ and } \frac{dq^{2N}}{dt} < (>)0. \tag{16}$$

If marginal costs are increasing in capital, output of firm 1 is increasing, and output of firm 2 is decreasing in the tax, and the opposite holds when marginal costs are decreasing in capital. Further, from (14) and (16) we know that:

$$\frac{dQ^N}{dt} > (<)0 \text{ if } \forall i c_x^i > (<)0 \text{ and } \left| c_x^1 \frac{dx^{1N}}{dt} \right| > \left| c_x^2 \frac{dx^{2N}}{dt} \Omega \right|;$$
$$\Omega = \frac{(1+\rho^1)\Pi_{11}^1}{(1+\rho^2)\Pi_{22}^2} > 0, \frac{dx^{1N}}{dt} < 0. \tag{17}$$

The effect of an increased tax on industry output therefore depends on the size of these two opposite effects. We have thus shown that the effect on output depends on the possible strategic effects of capital, the effect of capital on marginal costs, and the disparity of the two firms. From the properties of the SE, we also find that the more equal the two firms are, the more likely it is that they behave in the same manner, and that both firms' levels of output are decreasing in the tax.

4.3. THE EFFECT ON EMISSIONS: THE POSSIBILITY OF PERVERSE EFFECTS

The effect of the tax on emissions depends on both the effect on output and the effect on capital:

$$\frac{de^1}{dt} + \frac{de^2}{dt} = \sum_i \frac{\partial e^i}{\partial q^{iN}} \frac{dq^{iN}}{dt} + \frac{\partial e^i}{\partial x^{iN}} \frac{dx^{iN}}{dt}. \tag{18}$$

Generally, we would expect total emissions to be decreasing in the tax, but in our model this is not necessarily the case. The reasons for this are that output can be increasing in the tax, since marginal cost can be decreasing in capital, and capital and output can be increasing in the tax due to strategic effects.[6] Nevertheless, we can make some general observations. First, it is easy to see that if firm *i*'s output is decreasing and *i*'s capital is increasing in the tax, then *i*'s emissions are decreasing in the tax. Second, if this holds for both firms, then total emissions are also decreasing in the tax. Note that this can only occur when marginal costs are increasing in capital. Consequently, this is the only case when the sign of the effect of the tax on emission is clearly determined.[7] When marginal costs are decreasing in capital, output of at least one firm is increasing in the tax. Third, the magnitude of the second, imperfect competition correction part of the tax is decreasing in the emission reductions of the tax.

5. The Optimal Environmental Tax Revisited

In this section we analyse and summarise the results of the analysis, and investigate the optimal tax expressions. For analytical reasons, we restrict our attention to the case where total emissions are decreasing in the tax. This means that we focus on the numerator of the second part of the tax expressions. In section 4.2 we investigated what effect the tax has on equilibrium levels of output. The sign of this effect determines the sign of the second part of the optimal tax expression. Consequently, it also determines whether the optimal tax should be set lower or higher than the corresponding Pigouvian tax. The magnitude of the second part of the tax expression is determined by the magnitude of the effects on output and emissions. If the effects on output are small and the effects on emissions are large, then the second part will be small. For the closed loop and the Stackelberg case, the strategic- and leader effect will also be important. Therefore it is important to investigate the signs of these two effects separately.

For the Cournot open loop, output can only be decreasing in the tax for both firms when marginal costs are increasing in capital, and increasing in the tax when marginal costs are decreasing. Consequently, these are the only cases when the sign of the second part is unambiguously determined. In the first case, the optimal tax is lower than marginal environmental costs in order to not decrease output too much. In the second case, the tax is higher than marginal environmental costs in order to not increase output too much. The second case is similar to the model with endogenous market structure in Katsoulacos and Xepapadeas (1995), where the tax might exceed the marginal environmental costs in order to reduce output in terms of entering firms. The size of the second part of the optimal tax expression depends on the size of the changes in output and emissions. The interesting question then is what the sign will be when there are different effects on output. From the tax expression it is easy to see that the optimal tax is higher than marginal environmental costs if the effect on firm 1's output is larger than the effect on firm 2's output, and firm 2's market share is not so large that it balances this. The tax is also higher than marginal environmental costs if the effect on firm 1's output is smaller than the effect on firm 2's, but firm 1's market share is so large that it counteracts this effect. For simplicity, we now restrict our attention to, in capital, increasing marginal costs, but the analysis is easily extended to decreasing marginal costs. Suppose that output of firm 1 is increasing in the tax, and consequently output of firm 2 is decreasing in the tax. We can then derive restrictions on market shares that are sufficient for the sign of the second part of the tax expression to be determined.

PROPOSITION 1: *The optimal tax is lower than marginal environmental costs if* $s^{1N} \in (-\rho^2(1-\rho^2)^{-1}, (1-\rho^1)^{-1})$*, where* s^{1N} *is firm 1's market share.*

Proof: Let E denote the numerator of the second part of the tax expression, from (7) and (14) we then have that $E > 0$ iff $c_x^1 \frac{dx^{1N}}{dt}(\Pi_{11}^1)^{-1} < -c_x^2 \frac{dx^{2N}}{dt}(\Pi_{22}^2)^{-1}\Phi$, where $\Phi = \frac{1-s^{1N}(1-\rho^1)}{\rho^2+s^{1N}(1-\rho^2)}$. It is easy to show that $\Phi > 0$ if and only if $s^{1N} \in (-\rho^2(1-\rho^2)^{-1}, (1-\rho^1)^{-1})$. When Φ is positive the numerator is always positive, and the optimal tax is lower than marginal environmental costs.

Thus, when the market share for firm 1 is not very large or small the optimal tax is lower than marginal environmental costs, even when firm 1's output is increasing in the tax. The slopes of the reaction curves in output space give the restriction on the market share. The range of this restriction on the market share is decreasing in $|\rho^i|$. For small values of the $|\rho^i{'}s|$, the sign is unambiguously determined even when there is a large difference in the market shares. Intuitively this make sense, since a small value of $|\rho^i|$ implies that a firm is insensitive to changes of it's rival's output. However, the condition in proposition 1 is only a sufficient condition, and not a necessary. When Φ is negative, the sign of the numerator depends on the size of Φ, the size of the $\Pi_{ii}^i{'}s$, and the disparity in the costs imposed by the tax. In this case, it is easy to see that given that the output of firm 1 is increasing in the tax, firm 1's market share has to be small for the optimal tax to be lower than marginal environmental costs.[8]

For the Cournot closed loop, we first investigate the sign of the strategic effect of the optimal tax expression. The sign of this term depends on the effect the tax has on capital, and the effect capital has on marginal costs:

$$P'\frac{\partial q^{jN}}{\partial x^{iN}}\frac{dx^{iN}}{dt}\left(\frac{de^1}{dt}+\frac{de^2}{dt}\right)^{-1} > (<)0 \text{ if } \forall i c_x^i > (<)0 \text{ and } \frac{dx^{iN}}{dt} > 0. \quad (19)$$

When marginal costs are increasing in capital, and capital is increasing in the tax then the strategic effect is positive. The opposite holds when marginal costs are decreasing in capital. As in the open loop, output of both firms can only be decreasing in the tax when marginal costs are increasing in capital. However, the sign of the second part of the tax expression is not determined, because of the strategic effect. The strategic effect is always of the opposite sign for at least one firm, since capital of at least one firm must be increasing in the tax. The sign and size of the second part of the optimal tax in this case depends on the effect of the tax on output and emissions, the strategic effect, and the market shares. From the optimal tax expression it is easy to derive restrictions on the effects for the sign of the second part to be determined, and also to determine the magnitude. However, we can also derive restrictions on the market shares that are sufficient for the sign of the second part of the tax expression to be determined, given an assumption about the marginal cost function. Again, we restrict our attention to, in capital, increasing marginal costs.

PROPOSITION 2: *When both firms' levels of capital are increasing in the tax, the optimal tax is lower than marginal environmental costs if $s^{1N} \in (-\rho^2, 1+\rho^1)$ or*

$s^{1N} \in (1 + \rho^1, -\rho^2)$. When firm 1's level of capital is decreasing in the tax,[9] *the optimal tax is higher than marginal environmental costs if ($s^{1N} < (-\rho^2$ and $1 + \rho^1$)) or ($s^{iN} > (-\rho^2$ and $1 + \rho^1$)).*

Proof: Let E denote the numerator of the second part of the tax expression, from (9) and (14) we then have that $E > 0$ iff $c_x^1 \frac{dx^{1N}}{dt}(\Pi_{11}^1)^{-1} < -c_x^2 \frac{dx^{2N}}{dt}(\Pi_{22}^2)^{-1}\Phi$, where $\Phi = \frac{(1-\rho_1)-s_1^N}{s_1^N+\rho_2}$. It is easy to show that $\Phi > 0$ iff $s^{1N} \in (-\rho^2, 1 + \rho^1)$), or $s^{1N} \in (1 + \rho^1, -\rho^2)$ and the numerator is positive when $\Phi > 0$ and both firms' levels of capital are increasing in the tax. It is also easy to show that $\Phi < 0$ iff ($s^{1N} < (-\rho^2$ and $1 + \rho^1$)) or ($s^{1N} > (-\rho^2$ and $1 + \rho^1$)), and the numerator is negative when $\Phi < 0$ and firm 1's level of capital is decreasing in the tax.

In the first case the optimal tax is lower than marginal environmental costs when the market share for firm 1 is not very large or small. Again, the slopes of the reaction curves in output space give the restriction on the market share. The intuition behind this is similar to proposition 1, the difference being the presence of strategic effects, which affects the restrictions on the market share. Further, we cannot determine unambiguous limits on the market share. The first sufficient condition for Φ to be positive is a more restrictive condition than the condition for the open loop. The second sufficient condition can however be less restrictive. The range of the first restriction on the market share is decreasing in $|\rho^i|$, but the second restriction is increasing in $|\rho^i|$. Thus, when both firms' $|\rho^{i\prime}s|$ are either small or large, the sign is unambiguously determined even when there is a large difference in the market shares. When Φ is negative, the sign of the numerator depends on the size of Φ, the size of the $\Pi_{ii}^{i}{}'s$, and the disparity in costs imposed by the tax. For the second case of the proposition, the optimal tax is higher than marginal environmental costs if firm 1's market share is sufficiently low or high, where the slopes of the reaction curves give the restrictions. For example, when both firms are sensitive to changes in their rival's output, then even for a very small market share for firm 1, the second part of the tax expression is positive. When Φ is positive, the sign of the numerator depends on the size of Φ, the size of the $\Pi^i ii's$, and the disparity in the cost imposed by the tax.

For the Stackelberg case, we only investigate the sign of the so-called leader effect of the optimal tax expression:

$$P' \frac{\partial q^{2N}}{\partial x^{2N}} \gamma^2 \frac{dx^{1N}}{dt} \left(\frac{de^1}{dt} + \frac{de^2}{dt} \right)^{-1} > 0 \text{ if } \frac{dx^{1N}}{dt} > 0, \text{ and}$$
$$(c_x^2 < 0 \text{ and } \gamma^2 < 0) \text{ or } (c_x^2 > \text{ and } \gamma^2 > 0). \tag{20}$$

Let us assume that the leader's level of capital is increasing in the tax. When marginal costs are decreasing in capital and the firms' capital are strategic substitutes, then the leader effect of the tax is positive, and the opposite is true if marginal costs are increasing in the tax. The sign of the leader effect is reversed for strategic complements. Thus, the sign of this term depends on whether the levels of capital are strategic substitutes or complements. In addition, when marginal costs are increasing in capital, the leader effect and the strategic effect have the same sign only if the levels of capital are strategic complements. When marginal costs are decreasing in capital, the two effects have the same sign only if capital investments are strategic substitutes.

6. Conclusions

We have shown how information transmission influences firms' behaviour, and how that in turn influences the optimal tax. Previous models of optimal taxation have only considered the open loop case. To assume some sequential information transmission complicates the problem, since the regulator has to consider these effects when the tax is determined. We have also shown that an assumption has to be made about the effect of the strategic variable on marginal costs. As already discussed, it is not evident that the actions undertaken by the firms in order to reduce emissions necessarily result in higher marginal costs. From this it follows that an increased tax might increase emissions. Of course, this analysis also applies to non-optimal emission taxes, and is not merely of interest when analysing optimal taxes. The optimal tax consists of two parts: a Pigouvian tax, i.e. the marginal environmental costs, and a correction part due to imperfect competition. One interesting question is whether the second part is positive or negative, i.e. whether the optimal tax should be set higher or lower than the Pigouvian tax. We have shown that an essential assumption is the effect of the strategic variable on marginal costs, which in turn influences the effect of the tax on output. Moreover, the differences in market shares and the influence of the tax on the cost structure play an important role in the determination of the sign of the second part of the tax. The magnitude of the second part is decreasing in the total effect on emissions. We have also derived sufficient restrictions on the market shares to make the sign unambiguously determined. With information on the market shares and the slopes of the reaction curves, the regulator could at least know whether the Pigouvian tax is lower or higher than the optimal tax. The next task for the regulator is of course to determine the size of the second part of the tax expression. The first insight, although not new, is that the larger the effect on total emissions the less important the second part is. The second insight is that the size of the second part of the tax depends on the tax-effect on equilibrium levels of output, which in turn depends on the information transmission. Here, we showed that the strategic effect, for at least one firm, is always of the opposite sign to the direct effect.

We might argue that the analysis shows that fine-tuning of the tax is impossible in practice, since it requires detailed information about cost- and demand-conditions, information that is not always easily available to the regulator. It would perhaps therefore be tempting to argue that because of these information problems the regulator should disregard the effects of imperfect competition, and set the tax equal to marginal damage cost. However, since there is nothing in the analysis that shows that the effects of imperfect competition in general are small, this does not seem to be a good strategy for the regulator. In many of the cases, we have not been able to present clear results, since they depend on several, sometimes countervailing, effects. The effects of a tax depend on the cost- and demand-conditions. Therefore, it is necessary to analyse the tax and its effects under different specific assumptions about the cost- and demand-functions, if we wish to say anything about specific environmental regulations. In relation to this, an important area for future research is to compare an emission tax with other types of regulations.

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Notes

1. There are also other recent papers on emission abatement and environmental taxation in oligopolies. Damania (1996) investigates how emission taxes affect incentives for pollution abatement in an oligopoly supergame. Carraro and Soubeyran (1996) develop a model of optimal taxation in an oligopoly with environmental feedback. Katsoulacos and Xepapadeas (1996) develop a model of environmental taxation and R&D subsidies in an oligopoly with R&D spillovers.
2. However, several models of strategic environmental policy in international trade under imperfect competition have been developed (e.g. Barrett 1994; Simpson and Bradford 1994; Ulph 1991, 1996).
3. For strategic substitutes, an increase in q^j induces a decrease in q^i, and conversely for strategic complements (Bulow et al. 1985).
4. For a monopoly the optimal tax is $t = \frac{\partial D}{\partial e} + P'Q\frac{dQ}{dt}\left[\frac{\partial e}{\partial Q}\frac{dQ}{dt}+\frac{\partial e}{\partial x}\frac{dx}{dt}\right]^{-1}$, where $\frac{dQ}{dt} <(>) 0$ if $c_x >(<) 0$ and $\frac{dx}{dt} > 0$, i.e. the same formulation as in Barnett (1980). However, in our model, the sign of the second expression on the right hand side is only determined when marginal costs are increasing in capital. Then the expression is negative, and the optimal tax is lower than the corresponding Pigouvian tax. When marginal costs are decreasing in capital the effect of the tax on emissions is undetermined, and the sign of the second expression is undetermined.
5. Note that this always holds to be true for strategic complements.
6. For a monopoly we have that $de/dt < 0$ if $c_x > 0$. However, when $c_x < 0$ the sign is undetermined, since dQ/dt and dx/dt have the same sign. However, there are no strategic effects of the tax in this case.
7. Under an SE, total emissions are unambiguously decreasing in the tax when marginal costs are increasing in capital.
8. Since then from (14) $c_x^1(dx^{1N}/dt) < c_x^2(dx^{2N}/dt)\Pi_{11}^1/\Pi_{22}^2$, and Φ is decreasing in firm 1's market share.
9. Note that we have shown that this is only possible when the levels of capital are strategic substitutes, and in this case firm 1's output is increasing in the tax.

Appendix

Proof of Equation (7)

Differentiating equation (4) with respect to the emissions tax, we have:

$$[P - C_q^1]\frac{dq^{1N}}{dt} + [P - C_q^2]\frac{dq^{2N}}{dt} - [C_x^1 + r]\frac{dx^{1N}}{dt} - [C_x^2 + r]\frac{dx^{2N}}{dt} - \frac{\partial D}{\partial e}\left(\frac{de^1}{dt} + \frac{de^2}{dt}\right) = 0 \quad \text{(A1)}$$

Rewriting the first order conditions (5) and (6) we have that $(P - C_q^i) = -P'q^i + \frac{\partial e^i}{\partial q^i}$ and $-(C_x^i - r) = te_x^i$. Inserting these into (A1) gives us:

$$-P'\left[q^{1N}\frac{dq^{1N}}{dt} + q^{2N}\frac{dq^{2N}}{dt}\right] + \left(t - \frac{\partial D}{\partial e}\right)\left(\frac{de^1}{dt} + \frac{de^2}{dt}\right) = 0. \quad \text{(A2)}$$

Finally rearranging (A2) gives us (7). The proof of equation (9) and (11) is similar to this proof.

Proof of Equation (12)

Differentiating (8) with respect to the emission tax we have:

$$\begin{bmatrix} \Gamma_{ii}^i & \Gamma_{ij}^i & -e_x^i \\ \Gamma_{ji}^j & \Gamma_{jj}^j & -e_x^j \end{bmatrix} \begin{bmatrix} dx^{1N}/dt \\ dx^{2N}/dt \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}. \quad \text{(A3)}$$

The slope of the reaction curves in capital space can be written as $\gamma^i = -\frac{\Gamma_{ij}^i}{\Gamma_{ii}^i}$. Substituting this into (A3), we now have:

$$\begin{bmatrix} 1 & -\gamma^i & -e_x^i(\Gamma_{ii}^i)^{-1} \\ \gamma^j & 1 & -e_x^j(\Gamma_{jj}^j)^{-1} \end{bmatrix} \begin{bmatrix} dx^{1N}/dt \\ dx^{2N}/dt \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}. \quad \text{(A4)}$$

Using Cramer's rule and solving gives us the second expression in equation (12). Summing over the two firms, we have:

$$\frac{dx^{1N}}{dt} + \frac{dx^{2N}}{dt} = \frac{(1 + \gamma^2)e_x^1(\Gamma_{11}^1)^{-1} + (1 + \gamma^1)e_x^2(\Gamma_{22}^2)^{-1}}{1 - \gamma^1\gamma^2}, \quad \text{(A5)}$$

which is greater than zero by the restriction in (3).

Proof of Equation (14)

Differentiating (5) with respect to the tax, at the equilibrium levels of capital, we have:

$$\begin{bmatrix} \Pi_{ii}^i & \Pi_{ij}^i & -c_x^i(dx^{1N}/dt) \\ \Pi_{ji}^j & \Pi_{jj}^j & -c_x^j(dx^{2N}/dt) \end{bmatrix} \begin{bmatrix} dq^{1N}/dt \\ dq^{2N}/dt \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}. \quad \text{(A6)}$$

Using the fact that the slope of the reaction curves in output space can be written as $\rho^i = -\Pi^i_{ij}/\Pi^i_{ii}$, and substituting in (A6), we now have that:

$$\begin{bmatrix} 1 & -\rho^1 & -c^i_x(dx^{1N}/dt)(\Pi^i_{ii})^{-1} \\ -\rho^2 & 1 & -c^j_x(dx^{2N}/dt)(\Pi^j_{jj})^{-1} \end{bmatrix} \begin{bmatrix} dq^{1N}/dt \\ dq^{2N}/dt \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}. \tag{A7}$$

Using Cramer's rule and solving yields the first expression in (14). Summing over the two firms, we have:

$$\frac{dQ^N}{dt} = \frac{(1+\rho^2)c^1_x(dx^{1N}/dt)(\Pi^1_{11})^{-1} + (1+\rho^1)c^2_x(dx^{2N}/dt)(\Pi^2_{22})^{-1}}{1-\rho^1\rho^2}. \tag{A8}$$

Using the restrictions in (3), the condition in (14) is easy to establish.

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